UNSTEADY MOTION OF A CHARGED GAS WITH PARTICLE PRODUCTION

V. A. Levin

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 73-74, 1965

The expansion of a cloud of charged particles of a certain type within which particles of the same type are being produced is considered.

Let a cloud of charged particles of a single type with density $n_0(r)$ and velocity distribution $u_0(r)$ exist at time $t = 0$ in the region $r \ll R$. We assume that the rate of particle production is proportional to the particle density at a given point (as is true of avalanche processes) and that the particles produced are of the same type with zero initial velocity. Then the motion of the gas is described by the system of equations:

$$\frac{\partial n}{\partial t} + \frac{1}{r^{\nu-1}} \frac{\partial r^{\nu-1} nu}{\partial r} = \alpha n \qquad (\alpha = \alpha(t),\ g = \text{const}),$$
$$\frac{\partial nu}{\partial t} + \frac{1}{r^{\nu-1}} \frac{\partial r^{\nu-1} nu^2}{\partial r} = \frac{nq}{m} E + ng, \qquad \frac{1}{r^{\nu-1}} \frac{\partial r^{\nu-1} E}{\partial r} = 4\pi q n. \tag{1}$$

Here u is velocity, n density, E electric field strength, q particle charge, m particle mass, g mass force per unit mass (force of gravity), $\nu = 1, 2, 3$, respectively, for motions with plane, cylindrical and spherical symmetry.

We introduce the new independent variables (Lagrange variables)

$$\frac{\partial r_0}{\partial t} + u \frac{\partial r_0}{\partial r} = 0, \qquad \tau = t. \tag{2}$$

In these variables, system (1) has the form

$$\frac{\partial}{\partial \tau} r^{\nu-1} n \frac{\partial r}{\partial r_0} = \alpha(\tau)\, n r^{\nu-1} \frac{\partial r}{\partial r_0}, \qquad \frac{\partial r^{\nu-1} E}{\partial r_0} = 4\pi q n r^{\nu-1} \frac{\partial r}{\partial r_0},$$
$$\frac{\partial u}{\partial \tau} = -\alpha u + \frac{q}{m} E + g, \qquad \frac{\partial r}{\partial \tau} = u. \tag{3}$$

We integrate this system for the case $\nu = 1$.

The solution of this system, allowing for the initial data, is expressed as follows:

$$n = n_0 e^{\Omega(\tau)} \left[1 + \frac{4\pi q^2 n_0}{m} \int_0^\tau \frac{\operatorname{sh} \Omega(\xi)}{\alpha(\xi)} d\xi + \frac{du_0}{dx_0} \int_0^\tau e^{-\Omega(\xi)} d\xi \right]^{-1}, \tag{4}$$

$$u = u_0 e^{-\Omega(\tau)} + \frac{4\pi q^2}{m\alpha} \int_0^{x_0} n_0 dx_0 \operatorname{sh} \Omega(\tau) + g e^{-\Omega(\tau)} \int_0^\tau e^{\Omega(\xi)} d\xi, \tag{5}$$

$$x = x_0 + u_0 \int_0^\tau e^{-\Omega(\xi)} d\xi + \frac{4\pi q^2}{m} \int_0^{x_0} n_0 dx_0 \int_0^\tau \frac{\operatorname{sh} \Omega(\xi)}{\alpha(\xi)} d\xi + g \int_0^\tau e^{-\Omega(\xi)} \int_0^\xi e^{\Omega(\eta)} d\eta\, d\xi, \tag{6}$$

$$E = 4\pi q e^{\Omega(\tau)} \int_0^{x_0} n_0 dx_0 \qquad \left(\Omega(\zeta) = \int_0^\zeta \alpha\, d\eta\right). \tag{7}$$

We investigate the solutions obtained for the case $\alpha = \text{const}$. For the unknown quantities, we obtain

$$n = n_0 e^{\alpha\tau} \left[1 + \frac{4\pi q^2 n_0}{m\alpha^2} (\operatorname{ch} \alpha\tau - 1) + \frac{1}{\alpha} \frac{du_0}{dx_0} (1 - e^{-\alpha\tau})\right]^{-1}, \tag{8}$$

$$u = u_0 e^{-\alpha\tau} + \frac{g}{\alpha} (1 - e^{-\alpha\tau}) + \frac{4\pi q^2}{m\alpha} \operatorname{sh} \alpha\tau \int_0^{x_0} n_0 dx_0, \tag{9}$$

$$x = x_0 + \frac{u_0}{\alpha} (1 - e^{-\alpha\tau}) + \frac{4\pi q^2}{m\alpha^2} (\operatorname{ch} \alpha\tau - 1) \int_0^{x_0} n_0 dx_0 + \frac{g}{\alpha^2} (e^{-\alpha\tau} - 1 + \alpha\tau). \tag{10}$$

It is clear from (8) that as $\tau \to \infty$, irrespective of the initial conditions, n approaches the end value n_∞:

$$\lim_{\tau\to\infty} n = n_\infty = \frac{m\alpha^2}{2\pi q^2}. \tag{11}$$

Thus, for any n_0 and u_0, all space is occupied by particles whose density at each point is constant, while the limiting velocity distribution will be linear:

$$u_\infty = \alpha x \quad \text{when} \quad g = 0 . \tag{12}$$

We note in passing that if an avalanche of uncharged particles in a gravitational field is considered, it is clear that as $\tau \to \infty$ the velocity of all the particles in the avalanche becomes constant, although the number of particles set in motion increases without limit.

Let us see how the particle density in the cloud changes with time. For simplicity, we set n_0 = const, $u_0 = 0$. Then, two different cases are possible, depending upon the value of n_0.

1. When $n_0 > (1/2)n_\infty$, the particle density initially increases with increase in τ, reaches a maximum

$$n_{\max} = n_0\left(1 - \frac{n_\infty}{4n_0}\right)^{-1}, \qquad \tau_{\max} = -\frac{1}{\alpha}\ln\left(1 - \frac{1}{2}\frac{n_\infty}{n_0}\right),$$

and then decreases, approaching a limit.

2. When $n_0 \leq (1/2)n_\infty$ the particle density monotonically increases and approaches a limit. Finally, when $\tau \to \infty$, we obtain the steady-state solution

$$u = \alpha x, \qquad n = \frac{m\alpha^2}{2\pi q^2}, \qquad E = \frac{2m\alpha^2}{q}x \qquad (g = 0).$$

It can be shown that this solution is stable with respect to small perturbations of velocity and density.

In the case of a spherical cloud, it is not possible to integrate the system of equations in finite form, but the qualitative conclusions remain the same; in this case the density and limiting velocity distribution will be

$$n_\infty = \frac{m\alpha^2}{3\pi q^2}, \qquad u_\infty = \frac{\alpha r}{3}.$$

In conclusion, we note that these solutions may serve as an illustration of Hoyle's cosmological model [1, 2] and also have a bearing on the specific problem of the escape of a strongly nonisothermal plasma from the space in which it is formed.

REFERENCES

1. F. Hoyle, "A covariant formulation of the law of creation of matter," Monthly Notices Royal Astron. Soc., vol. 120, p. 256, 1960.

2. F. Hoyle and I. V. Narlikar, "A new theory of gravitation," Proc. Roy. Soc., A, vol. 282, p. 191, 1964.

14 December 1964 Moscow